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Mathematical analysis of PDE systems which govern fluid-structure interactive phenomena *†

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ABSTRACT: In this paper, we review and comment upon recently derived results for time dependent partial differential equation (PDE) models, which have been used to describe the various fluid-structure interactions which occur in nature. For these fluid-structure PDEs, this survey is particularly focused on the authors' results of (i) semigroup wellposedness, (ii) stability, and (iii) backward uniqueness.

Key Words: Partial differential equations, fluid-structure interaction, semigroups wellposedness, stabilization, backward uniqueness.

Contents

1. Introduction

In this survey article, we will present recently obtained results concerning the wellposedness and qualitative behavior of certain partial differential equation (PDE) systems, systems which have recently been derived to mathematically describe the phenomenon of an elastic body as it interacts (or is immersed) in a given fluid flow field. This "fluid-structure" PDE was originally proposed in [31] and subsequently in [27], the latter paper also providing an interesting review of the literature, as regards the various classes of such interactive PDE models. We defer to that reference for the history and development of these physically relevant PDE's, which are currently invoked to describe the coupling of fluid and solid; but

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by way of emphasizing the novelty of the fluid-structure problem under present consideration, we explicitly quote here the following statement from [27]:

> The majority of the references cited (in [27]) use solid models in lower spatial dimensions, e.g., one-dimensional beams interacting with two-dimensional fluids or two-dimensional plates interacting with three-dimensional fluids. Rigorous mathematical results are rare for fluid-structure interaction problems in which both the fluid and the solid occupy true spatial domains.

In short, there is less than a superfluidity of concrete results for fluid-structure PDE's in which both fluid and structure "enjoy" the same dimensionality. It is in this connection that we here present a compendium of our recent fluid-structure work. We should also make mention here of the recent and ongoing work of the authors in [14], who also provide a wellposedness and regularity theory for the fluid-structure dynamics (3)-(5) below, for linear and nonlinear variants of the model. The methodology of [14] is wholly different than that used to obtain the results posted in the present paper. In particular: As we shall see, the elimination of the pressure term $p(t, x)$ in (3)-(5) cannot be accomplished by an application of the classic Leray (or Helmholtz) Projector (see e.g., [19]), as is typically done with uncoupled fluid flow PDE models under the so-called "no-slip" boundary condition; the situation calls for a different approach. To this end, pressure $p(t)$ is eliminated in [14] by the means of equating the PDE (3)-(5) below with an appropriate variational relation; on the other hand, in the present survey the pressure term will be eliminated by identifying it as the solution of a certain elliptic boundary value problem, the forcing and boundary terms of which are composed of fluid and structure quantities.

2. The Stokes-Lamé system

Although we will ultimately announce and explain our results in the context of a more canonical fluid-structure model, a model whose relatively simple makeup will allow the reader to quickly digest our posted results without undue frustration at cumbersome notation, we will start by presenting the "physically relevant" PDE model which appears in the aforesaid [27]. For either (3)-(5) or the canonical (9)-(11), the geometry on which the fluid-structure interaction evolves will be the union $\Omega_f \cup \Omega_s$, where the "fluid domain" Ω_f is a bounded subset of $\mathbb{R}^n$, $n \geq 2$; likewise, the "solid domain" Ω_s of the geometry is a bounded subset of $\mathbb{R}^n$, which is moreover immersed in Ω_f. Also, we will denote $\nu(x)$ to be the unit outward normal with respect to Ω_f, and so *inward* with respect to Ω_s (see the Figure 1).

In presenting the fluid-structure model which explicitly appears in [27], it would behoove us to first recall the classical tensor operators which are invoked to mathematically describe the linear (Hookean) system of elasticity on the structural domain Ω_s (see e.g., [26]):

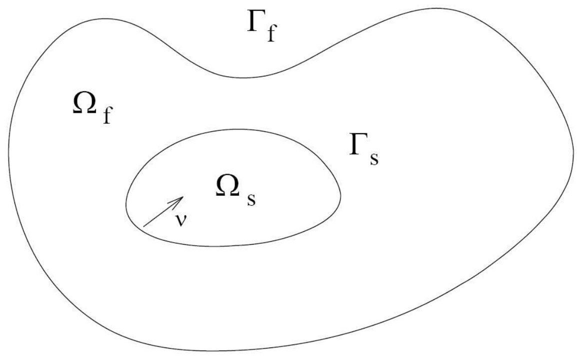

Figure 1: The Geometry of the Problem

1. For $\omega = [\omega_1, ..., \omega_n]$, the *strain tensor* $\{\epsilon_{ij}\}$ is given by
$$\epsilon_{ij}(\omega) = \frac{1}{2}\left(\frac{\partial \omega_j}{\partial x_i} + \frac{\partial \omega_i}{\partial x_j}\right), \ \ 1 \le i, j \le n. \tag{1}$$
2. Subsequently, the *stress tensor* is described by means of Hooke's Law:
$$\sigma_{ij}(\omega) = \lambda \left(\sum_{k=1}^{n} \epsilon_{kk}(\omega)\right) \delta_{ij} + 2\mu\epsilon_{ij}(\omega), \ \ 1 \le i, j \le n, \tag{2}$$
where $\lambda \ge 0$ and $\mu > 0$ are the so-called *Lamé's coefficients* of the system. Moreover, δ_{ij} denotes as usual the Kronecker delta; i.e., $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise.

With the geometry $\{\Omega_f, \Omega_s\}$ as described above (and again with unit normal ν exterior to Ω_f), and the stress-strain relations defined in (1)-(2), we are now in a position to describe the fluid-structure interactive PDE, which appears in [27] and which manifests a "boundary transmission condition" The variables (fluid) $u(t, x) = [u_1, u_2, ..., u_n]$ and (structure) $w(t, x) = [w_1, w_2, ..., w_n]$ satisfy

$$\text{PDEs} \begin{cases} u_t - \nabla \cdot \left(\nabla u + \nabla u^T\right) + \nabla p = 0 \quad \text{in } (0,T) \times \Omega_f \\ div(u) = 0 \quad \text{in } (0,T) \times \Omega_f \\ w_{tt} - div(\sigma(w)) + w = 0 \quad \text{in } (0,T) \times \Omega_s \end{cases} \tag{3}$$

$$\text{B.C.} \begin{cases} \left(\nabla u + \nabla u^T\right) \cdot \nu = \sigma(w) \cdot \nu + p\nu \quad \text{on } (0,T) \times \Gamma_s \\ u|_{\Gamma_f} = 0 \quad \text{on } (0,T) \times \Gamma_f \\ w_t|_{\Gamma_s} = u|_{\Gamma_s} \quad \text{on } (0,T) \times \Gamma_s \end{cases} \tag{4}$$

$$\text{I.C.} \quad [w(0), w_t(0), u(0)] = [w_0, w_1, u_0] \in \mathbf{H}. \tag{5}$$

Here $\mathbf{H}$ (eventually to be seen as constituting the finite energy space of wellposedness) is specified as

$$\mathbf{H} \equiv [H^1(\Omega_s)]^n \times [L^2(\Omega_s)]^n \times \mathcal{H}_f, \tag{6}$$

where fluid component space $\mathcal{H}_f \subset [L^2(\Omega_f)]^n$ is defined as follows:

$$\begin{aligned} Null(div) &= \left\{ f \in [L^2(\Omega_f)]^n : div(f) = 0 \right\}; \qquad (7) \\ \mathcal{H}_f &= \left\{ f \in Null(div) : \; [f \cdot \nu]_{\Gamma_f} = 0 \right\}. \qquad (8) \end{aligned}$$

We note immediately that the fluid component of the initial data is *not* that which is used in classical Navier-Stokes problems, in which the no-slip boundary condition is in play. In such uncoupled fluid flow problems, a function g is in the (Leray) space of wellposedness if $g \in Null(div)$ and $g \cdot \nu = 0$ on *all* of $\partial\Omega_f$. The adjustment of the fluid initial data is of course necessitated by the interaction on Γ_s between the fluid and structure dynamics.

As we said earlier, for the purpose of enhancing the readability of this survey, we will introduce a less busy-looking model, shorn of technical tensor notation, in order to illustrate our main fluid-structure results. However, we emphasize here, and will throughout, that the results to be announced for the canonical fluid-structure model (9)-(11) below are equally valid for the Stokes-Lamé PDE model (3)-(5).

3. The canonical fluid structure system

Our fluid-structure results will be announced and remarked upon in the context of the following interactive problem on the same geometry $\{\Omega_f, \Omega_s\}$ (again with unit normal ν exterior to Ω_f), in fluid variables $u(t,x) = [u_1, u_2, ..., u_n]$ and structure variables $w(t,x) = [w_1, w_2, ..., w_n]$:

$$\text{PDEs} \quad \begin{cases} u_t - \Delta u + \nabla p = 0 \quad \text{in } (0,T) \times \Omega_f \\ div(u) = 0 \quad \text{in } (0,T) \times \Omega_f \\ w - \Delta w + w = 0 \quad \text{in } (0,T) \times \Omega_s \end{cases} \tag{9}$$

$$\text{B.C.} \quad \begin{cases} \dfrac{\partial u}{\partial \nu} = \dfrac{\partial w}{\partial \nu} + p\nu \quad \text{on } (0,T) \times \Gamma_s \\ u|_{\Gamma_f} = 0 \quad \text{on } (0,T) \times \Gamma_f \\ w_t|_{\Gamma_s} = u|_{\Gamma_s} \quad \text{on } (0,T) \times \Gamma_s \end{cases} \tag{10}$$

$$\text{I.C.} \quad [w(0), w_t(0), u(0)] = [w_0, w_1, u_0] \in \mathbf{H}, \tag{11}$$

where the space of initial data $\mathbf{H}$ is the same as that given in (6), originally for the Stokes-Lamé system. So, instead of the unwieldy "divergence of the symmetric gradient" for the fluid and stress-strain tensors (for the structure), we have in place, in (9)-(11), the *vector-valued Laplacian*. That is, for $v(x) = [v_1(x), v_2(x), ..., v_n(x)]$,

$$\Delta v = [\Delta v_1, \Delta v_2, ..., \Delta v_n].$$

The results which will be announced for this PDE system (3)-(5) (and for the PDE (9)-(11)) address the three following issues: (i) *wellposedness*; (ii) *stability of solutions*; (iii) *backwards uniqueness*.

3.1. Semigroup wellposedness of the fluid-structure PDE. Even for the linear problem (9)-(11), the basic question of wellposedness was unresolved (see the quotation in the Introduction above). As we have mentioned, the work in [14] provides a theory of wellposedness, by means of considering an equivalent variational formulation. In this way, the authors of that work address the principal issue associated with the PDE (9)-(11) (or (3)-(5)); namely, a suitable elimination of the pressure term $p(t,x)$, as it appears in (9)-(11). In particular, since the boundary term $[u \cdot \nu]_{\Gamma_s}$ does not vanish on the fluid-structure interface Γ_s, the Leray Projector $\mathbb{P} : [L^2(\Omega_f)]^n \to [L^2(\Omega_f)]^n$ cannot be properly applied to both sides of the fluid equation in (9), as is conventionally done in classical Navier-Stokes Theory (see [19], [37]). Our approach here to eliminate the pressure variable is thus necessarily nonstandard (and very different than that taken in [14]). This approach was originally introduced in [6] and [8]. Our elimination of the pressure

is based upon the observation that if the pair $\{u, p\}$ solves the PDE in (9), then $p(t)$ necessarily solves the following elliptic problem on Ω_f, pointwise in time t:

$$\begin{cases} \Delta p(t) = 0 \ \text{ in } \Omega_f \\[2mm] p(t) = \dfrac{\partial u(t)}{\partial \nu} \cdot \nu - \dfrac{\partial w(t)}{\partial \nu} \cdot \nu \ \text{ on } \Gamma_s \\[2mm] \dfrac{\partial p(t)}{\partial \nu} = [\Delta u(t)] \cdot \nu \ \text{ on } \Gamma_f, \end{cases} \tag{12}$$

as can be verified directly by doing the necessary vector calculus operations. (We should also note that an observation of this type - i.e., the identification of the pressure function of *uncoupled* fluid flow to the solution of a certain elliptic boundary problem - was also made in the applied book [20] - although not exploited there to any particular end.) We can proceed then to use elliptic theory to write out $p(t)$ explicitly, in terms of the boundary data. In fact, we can define the "Dirichlet" and "Neumann" maps, D_s and N_s, respectively:

$$h = D_s g \Leftrightarrow \begin{cases} \Delta h = 0 \ \text{ in } \Omega_f; \\[2mm] h = g \ \text{ on } \Gamma_s; \\[2mm] \dfrac{\partial h}{\partial \nu} = 0 \ \text{ on } \Gamma_f; \end{cases} \qquad \psi = N_f \mu \Leftrightarrow \begin{cases} \Delta \psi = 0 \ \text{ in } \Omega_f; \\[2mm] \psi = 0 \ \text{ on } \Gamma_s; \\[2mm] \dfrac{\partial \psi}{\partial \nu} = \mu \ \text{ on } \Gamma_f. \end{cases} \tag{13}$$

By the known elliptic regularity (see e.g., [32]), we have for all r real,

$$\begin{aligned} D_s &\in \mathcal{L}(H^r(\Gamma_s), H^{r+\frac{1}{2}}(\Omega_f)); \\ N_f &\in \mathcal{L}(H^r(\Gamma_f), H^{r+\frac{3}{2}}(\Omega_f)). \end{aligned} \tag{14}$$

Through the agency of these elliptic maps, we can then write, for $0 < t < T$, the solution $p(t)$ of (12) - or what is the same, the pressure term in (9) for fixed t - as

$$p(t) = D_s \left\{ \left[\left(\frac{\partial u}{\partial \nu} - \frac{\partial w}{\partial \nu} \right) \cdot \nu \right]_{\Gamma_s} \right\} + N_f \left\{ [\Delta u \cdot \nu]_{\Gamma_f} \right\}. \tag{15}$$

Applying the gradient operator to both sides of this expression, we have then that, pointwise in time, the pressure term in (9) admits of the expression

$$\nabla p(t) = -G_1 w(t) - G_2 u(t), \tag{16}$$

where operators G_1 and G_2 are defined, respectively, by

$$G_1 w(t) \equiv \nabla D_s \left\{ \left[\frac{\partial w}{\partial \nu} \cdot \nu \right]_{\Gamma_s} \right\}; \tag{17}$$

$$G_2 u(t) \equiv -\nabla \left(D_s \left\{ \left[\frac{\partial u}{\partial \nu} \cdot \nu \right]_{\Gamma_s} \right\} + N_f \left\{ [\Delta u \cdot \nu]_{\Gamma_f} \right\} \right). \tag{18}$$

The point of constructing these "Green's maps" G_i, is that their invocation allows for the desired elimination of the pressure. To wit, in view of (17) and (18) the PDE system (3)-(5) can now be apparently written as

$$\frac{d}{dt} \begin{bmatrix} w(t) \\ w_t(t) \\ u(t) \end{bmatrix} = \mathcal{A} \begin{bmatrix} w(t) \\ w_t(t) \\ u(t) \end{bmatrix}; \quad \begin{bmatrix} w(0) \\ w_t(0) \\ u(0) \end{bmatrix} = \begin{bmatrix} w_0 \\ w_1 \\ u_0 \end{bmatrix} \in \mathbf{H}, \tag{19}$$

where $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$ is given by

$$\mathcal{A} \equiv \begin{bmatrix} 0 & I & 0 \\ \Delta - I & 0 & 0 \\ G_1 & 0 & \Delta + G_2 \end{bmatrix}. \tag{20}$$

Because of space limitations, we refrain here from giving a complete description of "the fluid-structure generator" $D(\mathcal{A})$, and instead refer the reader to the references [6] (for the canonical model fluid (9)-(11)) and [8] (for the Stokes-Lamé system (3)-(5)). It is enough here to know that if $[w_0, w_1, u_0] \in D(\mathcal{A})$, then there exists a "pressure" function $\pi_0 = \pi_0(w_0, u_0) \in [L^2(\Omega_f)]^n$ such that $[w_0, w_1, u_0, \pi_0]$

collectively satisfy the following properties:

$$
\begin{aligned}
&\text{(i) } w_0 \in [H^1(\Omega_s)]^n, \text{ with } \Delta w_0 \in [L^2(\Omega_s)]^n \\
&\qquad \text{(and so } \left.\frac{\partial w_0}{\partial \nu}\right|_{\Gamma_s} \in [H^{-\frac{1}{2}}(\Gamma_s)]^n\text{; see e.g., [25]);} \\
&\text{(ii) } w_1 \in [H^1(\Omega_s)]^n; \\
&\text{(iii) } u_0 \in \mathcal{H}_f \cap [H^1(\Omega_f)]^n, \text{ with } \Delta u_0 - \nabla \pi_0 \in \mathcal{H}_f; \\
&\text{(iv) } \left.\frac{\partial u_0}{\partial \nu}\right|_{\Gamma_s} \in [H^{-\frac{1}{2}}(\Gamma_s)]^n \text{ and } \left.\pi_0\right|_{\Gamma_s} \in H^{-\frac{1}{2}}(\Gamma_s); \\
&\text{(v) } \left.[(\Delta u_0)\cdot \nu]\right|_{\Gamma_f} \in H^{-\frac{3}{2}}(\Gamma_f); \\
&\text{(vi) } \left.\frac{\partial u_0}{\partial \nu}\right|_{\Gamma_s} = \left[\frac{\partial w_0}{\partial \nu} + \pi_0 \nu\right]_{\Gamma_s}; \\
&\text{(vii) } \left.u_0\right|_{\Gamma_f} = 0 \text{ on } \Gamma_f; \\
&\text{(viii) } \left.w_1\right|_{\Gamma_s} = \left.u_0\right|_{\Gamma_s} \text{ on } \Gamma_s, \text{ as elements of } [H^{\frac{1}{2}}(\Gamma_s)]^n.
\end{aligned}
\tag{21}
$$

For the operator $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$, it is natural to try to establish wellposedness by showing that one has an associated C_0-semigroup $\left\{e^{\mathcal{A}t}\right\}_{t\geq 0}$, possibly by the classical Lumer-Phillips Theorem (since one can straightforwardly integrate so as to verify a dissipation of energy for solutions of the system (9)-(11)). With the existence of $\left\{e^{\mathcal{A}t}\right\}_{t\geq 0}$, then one can quickly solve the abstract Cauchy problem (19), in the usual (weak) semigroup sense (see e.g., [13] or [35], p. 259). More importantly: owing to basic semigroup theory, one has wellposedness and continuity of the following map:

$$[w_0, w_1, u_0] \in D(\mathcal{A}) \Rightarrow [w(t), w_t(t), u(t)] \in C([0,T]; D(\mathcal{A})).$$

Combining this with the properties listed in (21) for $D(\mathcal{A})$, we conclude that if fluid-structure model $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$ generates a C_0-semigroup, then *smooth data* $[w_0, w_1, u_0]$ *gives rise to classical solutions of (9)-(11).*

Theorem 3.1 *(i) The operator* $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$*, defined by (20) is maximal dissipative, and so generates a* C_0*-semigroup of contractions on* $\mathbf{H}$*. Thus, for initial data* $[w_0, w_1, u_0] \in \mathbf{H}$*, there is a unique corresponding weak solution of (9)-(11) - or what is the same, a unique solution of the abstract Cauchy problem (19) - which satisfies the regularity,* $[w(t), w_t(t), u(t)] \in C([0,T]; \mathbf{H})$*. Moreover, we have wellposedness and continuity of the map,*

$$[w_0, w_1, u_0] \in \mathbf{H} \Rightarrow u \in L^2(0,T; [H^1(\Omega_f)]^n). \tag{22}$$

(ii) If in addition initial data $[w_0, w_1, u_0] \in D(\mathcal{A})$*, we have:*

$$\textit{(ii.a)} \quad [w(t), w_t(t), u(t)] \in C([0,T]; D(\mathcal{A})); \quad p \in C([0,T]; L^2(\Omega_f)), \tag{23}$$

with pressure p *being given by the expression*

$$\textit{(ii.b)} \quad p = D_s \left\{ \left[\left(\frac{\partial u}{\partial \nu} - \frac{\partial w}{\partial \nu} \right) \cdot \nu \right]_{\Gamma_s} \right\} + N_f \left\{ [\Delta u \cdot \nu]_{\Gamma_f} \right\}. \tag{24}$$

(iii) The Hilbert space adjoint $\mathcal{A}^* : D(\mathcal{A}^*) \subset \mathbf{H} \to \mathbf{H}$*, with* $D(\mathcal{A}^*) = D(\mathcal{A})$*, is likewise maximal dissipative.*

Combining (22) with the transmission boundary condition in (10) and Sobolev Trace Theory, we have moreover,

Corollary 3.2 *Given initial data* $[w_0, w_1, u_0] \in \mathbf{H}$*, the solution* $[w, w_t, u]$ *of (9)-(11) satisfies the following regularity (continuously):*

$$u|_{\Gamma_s} \in L^2(0,T; [H^{\frac{1}{2}}(\Gamma_s)]^n); \quad w_t|_{\Gamma_s} \in L^2(0,T; [H^{\frac{1}{2}}(\Gamma_s)]^n).$$

Some remarks concerning theorem 3.1

(1) The proof of wellposedness/semigroup generation is ostensibly classical, in that it is based on showing that $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$ is maximal dissipative, thereby allowing for an invocation of the Lumer-Phillips Theorem (see e.g., [35]). However, "the maximality part" of the proof, as it is given in [6], involves several nonstandard steps, the most delicate probably being a concise characterization of the *Range*$(\mathcal{A})$ (as we shall see in Section 4.3, zero is an eigenvalue of the Hilbert space adjoint $\mathcal{A}^* : D(\mathcal{A}^*) \subset \mathbf{H} \to \mathbf{H}$, and so *Range*$(\mathcal{A}) \subset \mathbf{H}$ only). In fact, it is shown in [6] that

$$Range(\mathcal{A}) = \left\{ [w_0^*, w_1^*, u_0^*] \in \mathbf{H} : \int_{\Gamma_s} w_0^* \cdot \nu d\Gamma_s = 0 \right\}. \tag{25}$$

One side of this containment is fairly immediate: In fact, if $[w_0^*, w_1^*, u_0^*] \in Range(\mathcal{A})$, there there exists $[w_0, w_1, u_0] \in D(\mathcal{A})$ such that

$$\mathcal{A} \begin{bmatrix} w_0 \\ w_1 \\ u_0 \end{bmatrix} = \begin{bmatrix} w_0^* \\ w_1^* \\ u_0^* \end{bmatrix}.$$

Using then the Definition of $\mathcal{A}$ in (20), the properties listed in (21) and the Divergence Theorem of Gauss, we have

$$\int_{\Gamma_s} w_0^* \cdot \nu d\Gamma_s = \int_{\Gamma_s} w_1 \cdot \nu d\Gamma_s = \int_{\Gamma_s} u_0 \cdot \nu d\Gamma_s = \int_{\Omega_f} div(u_0) d\Omega_f = 0. \tag{26}$$

It is thus the demonstration of the other set theoretic containment in (25) which represents the bulk of the effort.

(2) The extra L^2-regularity for the fluid component u in (22) is a direct manifestation of the underlying dissipation brought about by the fluid gradient, and hardly unexpected, given what is known for uncoupled Stokes flow (see [37]). Indeed, if we take the finite energy norm $\|\cdot\|_{\mathbf{H}}$ to be

$$\|[w_0, w_1, u_0]\|_{\mathbf{H}}^2 = \|\nabla w_0\|_{\Omega_s}^2 + \|w_0\|_{\Omega_s}^2 + \|u_0\|_{\Omega_f}^2 ;$$

and subsequently set

$$\mathcal{E}(t) \equiv \frac{1}{2} \|[w(t), w_t(t), u(t)\|_{\mathbf{H}}^2 , \quad \text{for } t > 0, \tag{27}$$

then, (i) a multiplication of the fluid equation in (9) by u, (ii) a multiplication of the structural component of (9) by w_t, and (iii) subsequent integration of the two respective relations in time and space, gives now: For all $t \geq s \geq 0$, the energy of the fluid-structure system (9)-(11) obeys the relation,

$$\mathcal{E}(s) = \mathcal{E}(t) - \int_s^t \|\nabla u\|_{\Omega_f}^2 \, d\tau, \tag{28}$$

which will imply (22) (with no loss of generality, we are assuming the initial data to be real-valued).

(3) For the original Stokes-Lamé system (3)-(5), one has a wellposedness result which is identical to Theorem 3.1. As we show in [8], we eliminate the pressure term $p(t, x)$ for the more convoluted PDE system (3)-(5), by the expedient of identifying it with the solution of an elliptic boundary value problem, wholly analogous to (12). In this way, we derive, in [8], an explicit Stokes-Lamé generator, whose appearance and structure is similar to that of $\mathcal{A}$ in (20). Moreover, we could establish the maximal dissipativity of said generator, in part by carefully characterizing its range, just as we sketched out in **(1)** for the generator $\mathcal{A}$ of the canonical (9)-(11). However, in [8], we actually proceed along different lines for the maximality argument: We show explictly the desired range condition $Range(\lambda I - \mathcal{A}) = \mathbf{H}$, for $\lambda > 0$, by a nonstandard usage of the Babuška-Brezzi Theorem (see e.g., [26]). One of the advantages of this approach, vis-á-vis that adopted in [6] (and alluded to in **(1)**), is that the former naturally gives rise to a finite element method (FEM) for approximating the solutions of fluid-structure interactive PDEs, a FEM that moreover does not necessitate the construction of divergence free basis functions, historically a thorny problem (see [4]).

(4) Modulo a change of sign in one of the Greens' map G_i, the explicit form of $\mathcal{A}^*$ is identical to that of $\mathcal{A}$.

4. Stability of the fluid-structure dynamics

In regards to the fluid-structure semigroup $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$, associated with the PDE model (9)-(11) (or (3)-(5)), we will address the following notions of stability: (i) *asymptotic* or *strong* stability, and (ii) *exponential* stability. Recall that given a generator $A : H \to H$ of a C_0-semigroup (with, say, H Hilbert), its corresponding semigroup $\left\{e^{At}\right\}_{t\geq 0} \subset \mathcal{L}(H)$ is said to be *strongly stable* if for all $v_0 \in H$, on has

$$\lim_{t\to\infty} e^{At} v_0 = 0. \tag{29}$$

In other words, if $v(t) \in C([0,T]; H)$ is the solution of the abstract Cauchy problem

$$v_t = Av, \ v(0) = v_0, \tag{30}$$

then $v(t) \to 0$ as $t \to \infty$. On the other hand, the semigroup $\left\{e^{At}\right\}_{t\geq 0}$ is *exponentially stable* if there exist positive constants C and ρ such that

$$\|v(t)\|_H = \left\|e^{At} v_0\right\|_H \leq Ce^{-\rho t} \|v_0\|_H \tag{31}$$

(so exponential is a stronger notion than strong stability).

Strong stability PDE results, as they haved appear in the literature, are typically a manifestation of "soft" functional analytical methods (see e.g., [28], [36], [39], [40]), and so tend to have short and elegant proofs which make for pleasurable reading. We should state here however, that one novelty of the strong stability problem for our fluid-structure generator $\mathcal{A}$ is that the resolvent operator $\mathcal{R}(\lambda; \mathcal{A})$ is *not* compact; this is shown outright in [6] and [8]. In consequence of this lack of compactness, our strong stability problem does not admit of a short and sweet solution, by means of Nagy-Foias-Fogel theory or the Lasalle Invariance Principle, these tools being used in the references above. (This lack of compactness for the resolvent has also been seen in structural acoustic flow PDE's, equations which also comprise a coupling of distinct dynamics across a boundary interface; see [5] and [33].)

On the other hand, uniform stability results for PDEs under feedback boundary control tend to have rather technical proofs which are driven by *a priori* inequalities and relevant identities. Moreover, geometry plays a definite role in boundary uniform stabilization; to obtain pluperfect results in this regard, results which require minimum geometrical restrictions, one typically has to appeal to certain PDE inequalities which are derived from a microlocal analysis of the associated localized problem (see e.g., the usage of the microlocal result from [30] in Section 4.2).

4.1. Strong stability of the fluid-structure PDE. One big advantage gained by having an explicit semigroup representation for the fluid-structure interaction (9)-(11) (or (3)-(5)), vis-à-vis the variational formulation in [14], is that one can directly undertake a spectral analysis of generator $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$, so as to glean qualitative information for solutions of (9)-(11). (In addition, to solve some given controllability problem, one can use the explicit form of $\mathcal{A}$ in (20) in order to clearly see what observability inequality, dual to controllability, must be generated.) By way of establishing conditions necessary for strong stability for (9)-(11), we performed just such an analysis for $\sigma(\mathcal{A}) \cap i\mathbb{R}$. In particular, we have the following spectral results which were proved in [6]:

Theorem 4.1 *(i) The point $\lambda = 0$ is an eigenvalue of both $\mathcal{A}$ and $\mathcal{A}^*$, with*

$$Null(\mathcal{A}) = Null(\mathcal{A}^*) = Span\left\{ \begin{bmatrix} \phi \\ 0 \\ 0 \end{bmatrix} \right\}, \tag{32}$$

where ϕ is the unique solution of the following elliptic boundary value problem:

$$\Delta\phi - \phi = 0 \quad in\ \Omega_s; \ \frac{\partial\phi}{\partial\nu} = \nu \quad on\ \Gamma_s,$$

where again $\nu(x)$ is the unit normal vector exterior to Ω_f (and so $nullity(\mathcal{A}) = nullity(\mathcal{A}^*) = 1$).

(ii.a) Consider the following(vector-valued) Dirichlet Laplacian eigenvalue problem with an additional Neumann boundary condition:

$$-\Delta\psi = \lambda\psi \ \ on\ \Omega_s; \ \ \psi|_{\Gamma_s} = 0 \ on\ \Gamma_s; \ \ \frac{\partial\psi}{\partial\nu} = \kappa\psi \quad on\ \Gamma_s, \tag{33}$$

where real $\lambda > 0$, real κ is non-zero, and both are otherwise unspecified. If this overdetermined eigenvalue problem admits only the trivial solution for all $(\lambda, \kappa) \in \mathbb{R}^+ \times (\mathbb{R}\diagdown\{0\})$, then $\sigma_p(\mathcal{A}) \cap i\mathbb{R} = \sigma_p(\mathcal{A}^*) \cap i\mathbb{R} = \{0\}$.

(ii.b) Suppose there are pairs $(\lambda, \kappa) \in \mathbb{R}^+ \times (\mathbb{R}\diagdown\{0\})$ for which there is a nontrivial solution ψ of (33) - which will be unique for the specified (λ, κ) (see [24]). Then $\sigma_p(\mathcal{A}) \cap i\mathbb{R}$ and $\sigma_p(\mathcal{A}^*) \cap i\mathbb{R}$ are at most countable, with each respective eigenvalue (and corresponding eigenfunction) on the imaginary axis explicitly identifiable.

(iii) $\sigma_c(\mathcal{A}) \cap i\mathbb{R} = \emptyset$ and $\sigma_r(\mathcal{A}) \cap i\mathbb{R} = \emptyset$. That is, there is no intersection of the continuous and residual spectra with the imaginary axis.

Remark 4.2 *Note that Theorem 4.1(ii) says, in essence, that the existence of purely imaginary eigenvalues depends on an affirmative answer to the following question: Is there any eigenfunction ψ of the (vector-valued) Laplacian operator on Ω_s with homogeneous Dirichlet boundary conditions, such that its normal derivative $\partial\psi/\partial\nu$ is a nonzero scalar multiple of the normal vector $\nu(x)$? If one can provide a negative response to this question, then $\sigma_p(\mathcal{A}) \cap i\mathbb{R}$ (and $\sigma_p(\mathcal{A}^*) \cap i\mathbb{R}$) consists of only the origin. Since it is wellknown that qualitative properties of spectra are often connected with geometry, one might naturally wonder if there are certain geometrical situations which give rise to the trivial solution in (33), thereby assuring that $\mathcal{A} : D(\mathcal{A}) \subset \mathbf{H} \to \mathbf{H}$ has no eigenvalues of the form ir, $r \in \mathbb{R}\diagdown\{0\}$. In fact, it is established in [6] that if the interactive boundary Γ_s is partially flat, then the overdetermined problem (33), for all parameters $(\lambda, \kappa) \in \mathbb{R}^+ \times (\mathbb{R}\diagdown\{0\})$, admits of only the trivial solution (and so Theorem 4.1(ii,a) obtains). This property* *is verified in [6] in the context of concrete examples involving flat or partially flat domains (e.g., parallelopipeds and circular cylinders) for which the Dirichlet Laplacian eigenpairs can be computed explicitly. Moreover, there plenty of other geometries Ω_s - in addition to when Γ_s is partially flat - which likewise yield that the solution ψ of (33) is trivial, and that therefore the conclusion of Theorem 4.1(ii.a) applies: Namely, $\sigma_p(\mathcal{A}) \cap i\mathbb{R} = \sigma_p(\mathcal{A}^*) \cap i\mathbb{R} = \{0\}$. (The details for this last assertion will be provided in a forthcoming paper.) On the other hand, it is also shown in [6] that there is at least one geometrical situation - i.e., a circular domain - for which the overdetermined problem (33) admits of countably many nontrivial solutions $\{\psi_n\}$, corresponding to their respective parameters $\{\lambda_n, \kappa_n\} \in \mathbb{R}^+ \times (\mathbb{R}\diagdown\{0\})$. Regardless of the geometrical situation however, Theorem 4.1(i) yields that $\lambda = 0$ is always an eigenvalue of $\mathcal{A}$ and $\mathcal{A}^*$, with respective eigenspaces each of dimension one.*

With a complete picture of $\sigma(\mathcal{A}) \cap i\mathbb{R}$, we are subsequently in a position to apply the wellknown spectral criterion for strong stability in [2] and [34], after "factoring out" the one-dimensional subspace $Null(\mathcal{A})$. (Recall that the resolvent $\mathcal{R}(\lambda; \mathcal{A})$ is not compact, so the aforesaid classical treatment for strong stability is inappropriate here.) In fact, we have the following:

Theorem 4.3 *(see [6]) If given initial data $[w_0, w_1, u_0]$ in (9)-(11) is in $[Null(\mathcal{A})]^\perp \subset$ $\mathbf{H}$, then the corresponding solution $[w, w_t, u] \in C([0,T]; [Null(\mathcal{A})]^\perp)$ decays strongly to the zero state. That is, $\lim_{t\to\infty} \|[w(t), w_t(t), u(t)]\|_{\mathbf{H}} = 0$.*

Remark 4.4 *Implicit in Theorem 4.3 is the statement that if initial data $[w_0, w_1, u_0] \in [Null(\mathcal{A})]^\perp$, the corresponding trajectory $[w(t), w_t(t), u(t)]$ stays in $[Null(\mathcal{A})]^\perp$ for all $t \geq 0$. This can be shown readily, using the fact, established in Theorem 4.1(i), that $\mathcal{A}$ and $\mathcal{A}^*$ share the same zero eigenfunction $[\phi, 0, 0]$, as given in (32).*

Remark 4.5 *The analogous result of strong stability is shown in [8] for solutions of the Stokes-Lamé system (3)-(5). Although the (purely spectral) modus operandi we undertook in [6] to prove Theorem 4.3 (and which we have outlined here) is surely applicable, we took a different approach in [8], which obviates the need to analyze continuous spectrum on $i\mathbb{R}$. Namely: if we denote $\mathcal{A}_L : D(\mathcal{A}_L) \subset \mathbf{H} \to \mathbf{H}$ to be the generator of the C_0-semigroup for the Stokes-Lamé PDE (3)-(5), then in [8] we establish the following strong limit:*

$$\lim_{\alpha\to 0^+} \sqrt{\alpha}\mathcal{R}(\alpha+i\beta; \mathcal{A}_L) \begin{bmatrix} w_0 \\ w_1 \\ u_0 \end{bmatrix} = 0 \quad \textit{for every } [w_0, w_1, u_0] \in \mathbf{H} \textit{ and every } \beta \in \mathbb{R}\diagdown\mathcal{K}, \tag{34}$$

where $\mathcal{K} \subset \mathbb{R}$ is a countable set specified in [8]. Combining the Banach-Steinhaus Theorem and a necessary estimate for the norm of $\mathcal{R}(\alpha + i\beta; \mathcal{A}_L)$ eventually gives *the inference that $i\beta \in \rho(\mathcal{A}_L)$. Since $\beta \in \mathbb{R}\diagdown\mathcal{K}$ was arbitrary, then we infer that $\sigma(\mathcal{A}_L)\cap i\mathbb{R}$ is at most countable. Having obtained information on $\sigma_p(\mathcal{A}_L)\cap i\mathbb{R}$ and $\sigma_p(\mathcal{A}_L^*)\cap i\mathbb{R}$ which is totally analogous to Theorem 4.1(i) and (ii), we can again appeal to [2] and [34] to make the conclusion of strong decay for solutions of the Stokes-elasticity system (3)-(5), when initial data is in $[Null(\mathcal{A}_L)]^{\perp}$. The notion of computing the strong limit (34) is drawn from the work in [18] and [38] (see also the earlier [16]), these papers being concerned with deriving resolvent criteria for strong stability, vis-à-vis the spectral criteria in [2] and [34].*

4.2. Uniform stability of the fluid-structure PDE. Given the seemingly strong dissipation coming from the gradient fluid component of the PDE system (9)-(11), one might be tempted to conjecture that solutions of this fluid-structure model actually decay at a *uniform*, not just *asymptotic*, rate, at least for initial data in $[Null(\mathcal{A})]^{\perp}$, and for appropriate geometrical configurations of the boundary interface Γ_s (see Theorem 4.1 and the subsequent Remark 4.2). But as we said earlier, the solution of the uniform stabilization problem for the PDE (9)-(11) - where here the "feedback stabilizer" is interpreted to be the gradient of the fluid component u - depends on generating a necessary *a priori* inequality. In particular, an inference that the solution $[w, w_t, u]$ of (9)-(11) obeys an inequality of the form (31), depends on a majorization of the energy $\mathcal{E}(T)$ strictly in terms of the dissipation. In the course of running the computations requisite for such an estimate, it becomes readily apparent that fluid dissipation alone is not enough to induce uniform decay for the entire system, comprising both fluid and structure. This situation is entirely analogous to that which prevails in [3], wherein a structural acoustic PDE, which consists of an acoustic wave equation, is coupled to a parabolic "elastic" equation which manifests strong Kelvin-Voight damping. As with our present PDE (9)-(11), the coupling of the two distinct structural acoustic PDE components is through means of a boundary interface. In [3], a result of uniform stabilization for these structural acoustic dynamics is given, in the case that additional feedback boundary dissipation is inserted into the wave component of the PDE system. Our point here in citing [3] is that, analogous to the previous structural acoustic situation, the strong property of uniform stability for the fluid-structure interaction (9)-(11) will require some feedback control mechanism on the structural as well as the fluid component.

In view of these remarks, we consider the following fluid structure PDE with additional boundary dissipation in one of the transmission conditions on Γ_s:

$$\text{PDEs} \quad \begin{cases} u_t - \Delta u + \nabla p = 0 \quad \text{in } (0,T)\times\Omega_f \\ div(u) = 0 \quad \text{in } (0,T)\times\Omega_f \\ w - \Delta w + w = 0 \quad \text{in } (0,T)\times\Omega_s \end{cases} \tag{35}$$

$$\text{B.C.} \quad \begin{cases} \dfrac{\partial u}{\partial \nu} = \dfrac{\partial w}{\partial \nu} + p\nu \quad \text{on } (0,T) \times \Gamma_s \\ u|_{\Gamma_f} = 0 \quad \text{on } (0,T) \times \Gamma_f \\ \left[w_t - \dfrac{\partial w}{\partial \nu} \right]_{\Gamma_s} = u|_{\Gamma_s} \quad \text{on } (0,T) \times \Gamma_s \end{cases} \tag{36}$$

$$\text{I.C.} \quad [w(0), w_t(0), u(0)] = [w_0, w_1, u_0] \in \mathbf{H} \tag{37}$$

In short, the boundary condition $w_t|_{\Gamma_s} = u|_{\Gamma_s}$ of (3)-(5) is replaced by $\left[w_t - \frac{\partial w}{\partial \nu} \right]_{\Gamma_s} = u|_{\Gamma_s}$. This latter expression induces an additional structural dissipation: In fact, analogous to the regularity result in Theorem 3.1, we have the following continuous map for solutions of the PDE (35)-(37) (see [7]):

$$\begin{aligned} &[w_0, w_1, u_0] \in \mathbf{H} \Rightarrow \\ &[w, w_t, u] \in C([0,T]; \mathbf{H}), \;\; u \in L^2(0,T; [H^1(\Omega_f)]^n), \;\; \frac{\partial w}{\partial \nu} \in L^2(0,T; L^2(\Gamma_s)). \end{aligned} \tag{38}$$

In particular, to justify the asserted L^2-in time regularity in (38), we can invoke a simple energy method, as was employed for relation (28), so as to have

$$\mathcal{E}(s) = \mathcal{E}(t) - \int_s^t \|\nabla u\|_{\Omega_f}^2 \, d\tau - \int_s^t \left\| \frac{\partial w}{\partial \nu} \right\|_{\Gamma_s}^2 d\tau, \tag{39}$$

where the "energy" function $\mathcal{E}(t)$ associated with (35)-(37) is as defined in (27). Given this dissipative relation, then to establish the uniform decay estimate (31) for the fluid-structure model, it suffices from a classic argument (see [12]) to show the following upperbound for the energy, for some positive constant C_T:

$$\mathcal{E}(T) \leq C_T \left(\int_0^T \|\nabla u\|_{\Omega_f}^2 \, dt + \int_0^T \left\| \frac{\partial w}{\partial \nu} \right\|_{\Gamma_s}^2 dt \right). \tag{40}$$

One can show readily that the extra structural dissipation removes the zero eigenvalue, as well as the purely imaginary eigenvalues that could arise (see Theorem 4.1), and so consequently solutions of (35)-(37) decay strongly for all $[w_0, w_1, u_0] \in \mathbf{H}$ by virtue of the spectral criteria in [2] and [34]. But we have in fact the stronger result: By establishing the *a priori* inequality (40) we have,

Theorem 4.6 *(see [7]) For given initial data $[w_0, w_1, u_0] \in \mathbf{H}$, the solution of the fluid-structure PDE (35)-(37) decays exponentially in time. That is to say, there exist positive constants C and ρ such that the solution $[w, w_t, u]$ of (35)-(37) exhibits the decay rate*

$$\|[w(t), w_t(t), u(t)]\|_{\mathbf{H}} \leq C e^{-\rho t} \|[w_0, w_1, u_0]\|_{\mathbf{H}} \quad \textit{for all } 0 \leq t \leq T. \tag{41}$$

Remark 4.7 *The proof of this result in [7] involves, in part, a "multiplier method" which to some extent is a vector-valued version of that carried out for boundary-controlled (and scalar-valued) wave equations; see e.g., [39], which follows the Lyapunov method-based papers [17], [28]. Note that a key feature of Theorem 4.6 is the validity of the decay rate (41) with* no *geometrical assumptions being imposed upon the boundary interface Γ_s. The "big gun" which allows for this generality is the following microlocal result, which provides for the treatment of (historically troublesome) boundary integrals involving the tangential derivative $\partial w/\partial t$, these occurring in the course of establishing (40), via said multiplier method:*

Lemma 4.8 *(See [30]) Let $\epsilon > 0$ be arbitrarily small. Let z solve an arbitrary second-order hyperbolic equation with smooth space-dependent coefficients on $Q_T \equiv (0,T) \times \Omega$, where $\Omega \subset \mathbb{R}^n$ is a smooth bounded domain. Then if Γ_* is a smooth connected segment of boundary $\partial\Omega$, we have the estimate*

$$\begin{aligned}\int_{\epsilon}^{T-\epsilon}\int_{\Gamma_*}\left(\frac{\partial z}{\partial \tau}\right)^2 dtd\partial\Omega \quad &\leq \quad C_T\left(\int_0^T\int_{\Gamma_*} z_t^2 dtd\partial\Omega + \int_0^T\int_{\partial\Omega}\left(\frac{\partial z}{\partial \nu}\right)^2 dtd\partial\Omega\right) \\ &+C_T\left(\|z\|^2_{H^{\frac{1}{2}+\epsilon_0}(Q_T)}\right), \qquad (42)\end{aligned}$$

where parameters $\epsilon, \epsilon_0 > 0$ are arbitarily small.

Applying this ΨDO trace result to the vector-valued function $\frac{\partial w}{\partial \tau}\big|$ at the tail end of our multiplier method, we eventually arrive at the preliminary estimate

$$\mathcal{E}(T) \leq C_T\left(\int_0^T \|\nabla u\|^2_{\Omega_f}\, dt + \int_0^T \left\|\frac{\partial w}{\partial \nu}\right\|^2_{\Gamma_s} dt\right) + \mathsf{l.o.t.}(w, w_t), \qquad (43)$$

where $\mathsf{l.o.t.}(w, w_t)$ denote "lower order terms", or measurements of $\{w, w_t\}$ in a (spatial) topology lower than that of the finite energy $\mathbf{H}$. Subsequently, a compactness-uniqueness argument (by contradiction), which uses the classic Holmgren's result for the uniqueness of the continuation, removes these polluting lower order terms, so as to establish (40), and so then the estimate (41).

Remark 4.9 *One has the exact analogue of the exponential stability result for the Stokes-Lamé system (3)-(5) with inserted structural feedback dissipation. In particular, if the transmission condition $[w_t]_{\Gamma_s} = u|_{\Gamma_s}$ is replaced by $[w_t - \sigma(w)\cdot\nu]_{\Gamma_s} = u|_{\Gamma_s}$, then as we noted for the canonical model (35)-(37), the corresponding solutions of the Stokes-Lamé PDE, with extra Neumann dissipative feedback, decay exponentially in time. This work has been done in [10]. Our modus operandi in this preprint is very much as we detailed for the canonical (35)-(37): In [10] we invoke a multiplier method to establish the energy inequality needed for exponential stability, and which is wholly analogous to (40). Namely, if $[w, w_t, u]$ solves* *the aforesaid Stokes-Lamé system with Neumann boundary dissipation, we must establish the following estimate to infer exponential decay:*

$$\|[w(T), w_t(T), u(T)]\|_{\mathbf{H}}^2 \leq C_T \left(\int_0^T \left\| \nabla u + \nabla u^T \right\|_f^2 dt + \int_0^T \|\sigma(w)\nu\|_{\Gamma_s}^2 dt \right). \quad (44)$$

In the work [10], the known energy identities for the Lamé system of elasticity are put to good use (see e.g., [1], [15], [22]). At some point in the course of this method, there is the need to estimate $\|D_\tau w\|_{L^2(0,T;L^2(\Gamma_s))}$*, similar to the situation we outlined above for (35)-(37). By way of dealing with this tangential gradient, we invoke the trace estimate in [23] (see also [42]) for solutions of the system of elasticity, this estimate being the natural descendant of the wave equation estimate (43). Eventually, by the means we have sketched out, we reach a point in [10] at which we derive the following estimate:*

$$\begin{aligned}\|[w(T), w_t(T), u(T)]\|_{\mathbf{H}}^2 \quad \leq \quad & C_T \left(\int_0^T \left\| \nabla u + \nabla u^T \right\|_f^2 dt + \int_0^T \|\sigma(w)\nu\|_{\Gamma_s}^2 dt \right) \\ & + \mathsf{l.o.t.}(w, w_t),\end{aligned}$$

where again, $\mathsf{l.o.t.}(w, w_t)$ *denotes polluting lower order terms. As we did for the proof of Theorem 4.6, we wish to complete the derivation of estimate (44) by invoking a compactness-uniqueness argument. To make the uniqueness part of this argument (by contradiction) work, we invoke the unique continuation result in [21] for systems of elasticity. (Note that compared to the classic Holmgren's theorem, the result in [21] is relatively "state of the art".)*

5. Backwards uniqueness of the fluid-structure dynamics

The Remark 4.9 is slightly deceptive, as it conveys the impression that by freely following the game plan provided in [7] for the uniform stabilization of the canonical fluid-structure PDE (35)-(37), we readily obtained in [10], with nary a hitch, the like result for the physical PDE model (3)-(5) with structural boundary dissipation. Certainly, the *modus operandi* of [7] is directly applicable in [10], as we have outlined above, but in the course of proving exponential decay for the structurally damped Stokes-Lamé system, one encounters an issue not seen for the canonical (35)-(37). Namely: The classic Holmgren's uniqueness argument is invoked in [7] to essentially show that the canonical model (9)-(11) with overdetermined homogeneous boundary conditions necessarily implies the trivial solution; in the language of control theory, this is a property of *approximate controllability* (by duality); see e.g., [41]. On the other hand, in proving the analogous approximate controllability for the Stokes-Lamé PDE (3)-(5) in [10], there is recourse, as we said, to the new unique continuation result [21], which however provides the desired uniqueness property only at some time T_0, say, and which is not necessarily the origin.

In short, to complete the proof of uniform stabilization for the Stokes-Lamé system, we had to ascertain that these fluid-structure models satisfy the so-called "backwards-uniqueness" property. This can be stated as follows:

Backward-Uniqueness Property Given Banach space X, let $A : D(A) \subset X \to X$ be a C_0-semigroup. Then $\left\{e^{At}\right\}_{t\geq 0} \subset \mathcal{L}(X)$ is said to satisfy the *backward-uniqueness* property if,

$$\text{whenever } e^{AT_0}x_0 = 0 \text{ for some } T_0 > 0 \text{ and } x_0 \in X, \text{ then } x_0 = 0. \tag{45}$$

In trying to establish this property for the fluid-structure model (9)-(11), the problem may be framed thus: The semigroup associated with the uncoupled wave equation satisfies (45) due to the underlying conservation of energy (i.e., the wave C_0-semigroup is in fact a group); moreover, the Stokes operator (posed on the usual Leray space) generates an *analytic* semigroup, and so is associated with the property (45). Does now the fluid-structure model (9)-(11) retain this property, a property enjoyed by its two constitutive parts? It is our good fortune that we can give a clean answer to this question.

Theorem 5.1 *(see [11]) The fluid-structure semigroup* $\left\{e^{\mathcal{A}t}\right\}_{t\geq 0} \subset \mathcal{L}(\mathbf{H})$ *obeys the backward-uniqueness property (45). That is to say, if the solution* $[w, w_t, u] \in C([0,T]; \mathbf{H})$ *of (9)-(11), corresponding to initial data* $[w_0, w_1, u_0] \in \mathbf{H}$*, satisfies* $[w(t_0), w_t(t_0), u(t_0)] = \vec{0}$ *for some* $0 < t_0 \leq T$*, then necessarily* $[w_0, w_1, u_0] = 0$.

Remark 5.2 *It hardly needs to be said that the backwards uniqueness property has also been established for the Stokes-Lamé* C_0*-semigroup* $\mathcal{A}_L : D(\mathcal{A}_L) \subset \mathbf{H} \to \mathbf{H}$ *which abstractly models (3)-(5) (else the paper [10] could not rightly exist); this is shown in [9]. The proofs in both [11] and [9] center on establishing that fluid-structure semigroups* $\mathcal{A}$ *and* $\mathcal{A}_L$ *obey the uniform norm estimate (46), in order to apply the following operator theoretic result:*

Theorem 5.3 *(see Theorem 3.1 of [29]) Let* A *be the infinitesimal generator of a* C_0*-semigroup in a Banach space* X*. Assume there exists constants* $\theta \in (\frac{\pi}{2}, \pi)$*,* $R > 0$ *and* C*, such that*

$$\left\|(A - re^{\pm i\theta})^{-1}\right\| \leq C, \tag{46}$$

for all $r \geq R$*. Then* A *obeys the backwards uniqueness property (45).*

Although the Theorem 5.3 is a relatively simple-looking, almost elegant, statement, the actual verification that the respective fluid-structure semigroups $\mathcal{A}$ and $\mathcal{A}_L$ satisfy the uniform estimate (46), at least for some (judiciously chosen) rays on the left complex plane, is a fairly complicated task. It is a task involving the appropriate estimation of ungainly-looking, static fluid-structure PDE's, which necessarily have real and complex parts. We beg off from providing the unpalatable technical details, and instead refer the reader to [11] and [9].

References

1. F. Alabau and V. Komornik, "Boundary observability, controllability and stabilization of linear elastodynamic systems", in *Contemporary Mathematics 209: Optimization Methods in Partial Differential Equations*, Amer. Math. Soc., Providence, RI (1997), pp. 1-8.

2. W. Arendt and C. J. K. Batty, *Tauberian theorems and stability of one parameter semigroups*, Trans. Amer. Math. Soc. **306** No. 8 (1988), pp. 837-852.

3. G. Avalos, *The Exponential Stability of a Coupled Hyperbolic/Parabolic System Arising in Structural Acoustics*, Abstract and Applied Analysis, Vol. 1, no. 2 (1996), pp. 203–217.

4. G. Avalos and M. Dvorak, "A divergence free FEM formulation for fluid-structure interactions", preprint (2007).

5. G. Avalos and I. Lasiecka, *The strong stability of a semigroup arising from a coupled hyperbolic/parabolic system*, Semigroup Forum, Vol. 57 (1998), pp. 278-292.

6. G. Avalos and R. Triggiani, "The coupled PDE system arising in fluid/structure interaction, Part I: explicit semigroup generator and its spectral properties", Contemporary Mathematics 440: Fluids and Waves, American Mathematical Society, Providence Rhode Island (2007), pp. 15-54.

7. G. Avalos and R. Triggiani, "Uniform stabilization of a coupled PDE system arising in fluid-structure interaction with boundary dissipation at the interface", to appear in Discrete and Continuous Dynamical Systems.

8. G. Avalos and R. Triggiani, "Wellposedness and stability analysis of a coupled Stokes-Lamé system, as a PDE model of certain fluid-structure interactions, preprint (2007).

9. G. Avalos and R. Triggiani, "The backwards uniqueness property for the C_0- semigroup associated with a Stokes-Lamé partial differential equation system, preprint (2007).

10. G. Avalos and R. Triggiani, "concerning the interactive boundary feedback stabilization of a coupled Stokes-Lamé PDE System", preprint (2007).

11. G. Avalos and R. Triggiani, "Backwards uniqueness of the S.C. semigroup arising in parabolic-hyperbolic fluid-structure interaction, to appear in Journal of Differential Equations.

12. A. V. Balakrishnan, *Applied Functional Analysis and Applications*, Second Edition, Springer-Verlag, New York (1981).

13. J. M. Ball, "Strongly continuous semigroups, weak solutions and the variation of constants formula", Proceedings of American Math Society, **63**, pp. 370-373 (1977).

14. V. Barbu, Z. Grujić, I. Lasiecka and A. Tuffaha, "Weak and strong solutions in nonlinear fluid-structure interactions", Contemporary Mathematics 440: Fluids and Waves, American Mathematical Society, Providence Rhode Island (2007), pp. 55-82.

15. Assia Benabdallah and Irena Lasiecka, "Exponential decay rates for a full von Karman systemn of dynamic thermoelasticity", Journal of Differential Equations, **160** (2000), pp. 51-93.

16. K. N. Boyadzhiev and N. Levan, Strong stability of Hilbert space contraction semigroups, Stud. Sci. Math. Hung. 30 (1995), pp. 162-182.

17. G. Chen, "A note on the boundary stabilization of the wave equation", SIAM J. Control Optim., **19** (1981), pp. 106-113.

18. R. Chill and Y. Tomilov, *Stability of operator semigroups: ideas and results*, preprint (2005).

19. P. Constantin and C. Foias, *Navier-Stokes Equations*, The University of Chicago Press, Chicago (1989).

20. C. R. Doering and J. D. Gibbon, *Applied Analysis of the Navier-Stokes Equations*, Cambridge Texts in Applied Mathematics, Cambridge University Press, New York (1995).

21. M. Eller, V. Isakov, G. Nakamura, and D. Tataru, "Uniqueness and stability in the Cauchy Problem for Maxwell's and elasticity systems", Nonlinear partial differential equations and their applications, Collège de France, Vol. XIV (Paris, 1997/1998), Stud. Math. Appl., 31, North-Holland, Amsterdam (2002), pp. 329-349.

22. M. A. Horn, "Implications of sharp trace regularity results on boundary stabilization of the system of linear elasticity", Jornal of Mathematical Analysis and Applications, **223** (1998), pp. 126-150.

23. M. A. Horn, "Sharp trace regularity for the solutions of the equations of dynamic elasticity", J. Math. Systems, Estim. Control **8** (2), 217-219.

24. V. Isakov, *Inverse Problems for Partial Differential Equations*, Springer, New York (1998).

25. B. Kellogg, "Properties of solutions of elliptic boundary value problems", in *The Mathematical Foundations of the Finite Element Method with Applications to Partial Differential Equations*, Edited by A. K. Aziz, Academic Press, New York (1972), pp. 47-81.

26. S. Kesavan, *Topics in Functional Analysis and Applications*, John Wiley & Sons, New York (1989).

27. Q. Du, M. D. Gunzburger, L. S. Hou and J. Lee, "Analysis of a linear fluid-structure interaction problem", Discrete and Continuous Dynamical Systems, Volume 9, No. 3 (May 2003), pp. 633-650.

28. J. Lagnese, "Decay of solutions of wave equations in a bounded region with boundary dissipation", J. Differential Equations **50** (1983), pp. 163-182.

29. I. Lasiecka, R. Triggiani, and M. Renardy, "Backwards uniqueness for thermoelastic plates with rotational forces", Semigroup Forum, Vol. 62 (2001), pp. 217-242.

30. I. Lasiecka and R. Triggiani, "Uniform stabilization of the wave equation with Dirichlet or Neumann feedback control without geometrical conditions", Appl. Math. Optim. 25 (1992), pp. 189-224.

31. J.L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, Paris (1969).

32. J.L. Lions and E. Magenes, *Non-homogeneous boundary value problems and applications*, Vol. I, Springer-Verlag (1972).

33. W. Littman and B. Liu, "On the spectral properties and stabilization of acoustic flow", IMA, University of Minnesota, *IMA Preprint Series #1436*, (November, 1996).

34. Y. I. Lyubich and V. Q. Phong, *Asymptotic stability of linear differential equations in Banach spaces*, Studia Matematica, LXXXVII (1988), pp. 37-42.

35. A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, New York (1983).

36. J. Quinn and D. L. Russell, *Asymptotic stability and energy decay rates for solutions of hyperbolic equations with boundary damping*, Proc. Roy. Soc. Edinburgh Sect. A **77** (1977), pp. 97-127.

37. R. Temam, *Navier-Stokes Equations*, North Holland, Amsterdam (1978).

38. Y. Tomilov, *A resolvent approach to stability of operator semigroups*, J. Operator Theory **46** (2001), pp. 63-98.

39. R. Triggiani, *Wave equation on a bounded domain with boundary dissipation: an operator approach*, Journal of Mathematical Analysis and Applications **137** (1989), pp. 438-461.

40. J. Zabczyk, *Stabilization of boundary control systems*, in “International Symposium on Systems Optimization and Analysis, December 1978”, Lecture Notes in Control and Information Sciences (A. Bensoussan and J. L. Lions, Eds.), Vol. 14, Springer-Verlag, New York/Berlin.

41. J. Zabczyk, *Mathematical Control Theory: An Introduction*, Birkhäuser, Boston (1992).

42. I. Lasiecka and R. Triggiani, *Uniform stabilization of a shollow shell model with nonlinear boundary feedback.* J. Math Analysis and applications, vol. 269 (2002), pp. 642-688.

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